United Kingdom and Ireland Programming Contest 2019

Problems

Problems are not ordered by difficulty.
Do not open before the contest has started.

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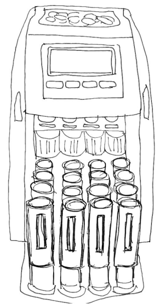
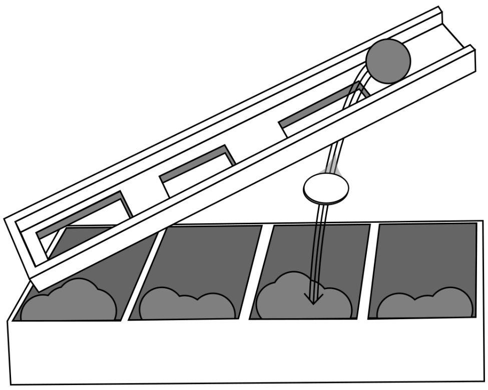
Problem A
Automatic Accountant



The bank you work in has purchased an advanced technological solution to the problems it has with counting money deposited by clients. The machine works by running each individual coin along a sloped track. At every integer multiple of centimetres along, starting from 1cm, there is a slot in the track with a bucket underneath.



The slot will allow a coin to fall in, if the thickness of the coin (in millimetres) is **less than or equal to** the width of the slot (also in millimetres), and the mass of the coin (in grams) is **greater than or equal to** the trigger mass of the slot (also in grams).

Since the slots are spaced $1cm$ apart centre-to-centre, and since there can be a large number of coins (or other metal shapes) inserted, the amount of wear on the track will depend on total distance travelled by all coins.

Given a list of the coins that will be deposited, what total distance will they travel, in centimetres?

Input

The input consists of:

- one line containing the number of slots, s ($1 \leq s \leq 10^5$).
- s further lines, the ith line containing the width of a slot in millimetres and trigger mass in grams of the ith slot, a_i and b_i respectively ($1 \leq a, b \leq 10^5$).
- one line containing the integer c ($1 \leq c \leq 10^5$), the number of coins.
- c further lines, the jth line containing the thickness in millimetres and mass in grams of the jth coin, u_i and v_i respectively ($1 \leq u, v \leq 10^5$).

It is guaranteed that every coin will be able to fall into at least one slot.

Output

Output the total distance in centimetres travelled by coins.

Sample Input 1

```
1
10 10
1
5 15
```

Sample Output 1

```
1
```

Sample Input 2

```
3
2 2
1 3
1 1
2
2 2
1 1
```

Sample Output 2

```
4
```

Sample Input 3

```
3
2 2
1 3
1 1
2
2 2
1 2
```

Sample Output 3

```
2
```

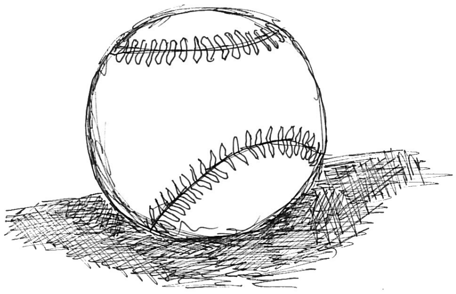
Sample Input 4

```
5
2 3
2 2
4 2
4 5
10 1
5
1 5
2 1
4 2
5 3
5 5
```

Sample Output 4

```
19
```

Problem B
Ballpark Estimate



Giving the right level of detail is an important skill for efficient communication. Sometimes, only the high-level message matters.

For example, whenever a person asks for a number, often they just want an estimate. If the value is in the millions, they do not need to know the precise number of hundreds and tens. Likewise, if the value is in the billions, they do not necessarily care about little things like millions.

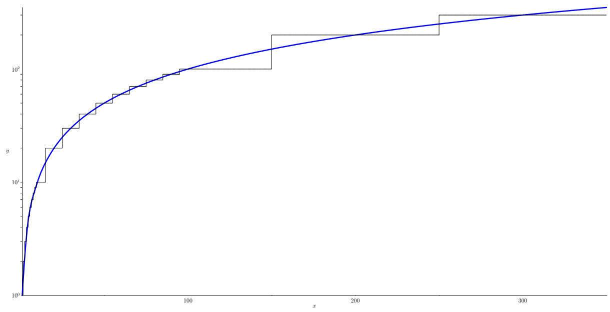


Figure B.1: Illustration of ballpark figures versus actual figures, as a log chart.

Given a (possibly very large) number, print its numerically closest representation that has only one digit other than trailing zeroes.

The closeness of the representation r of a number n is defined by $\text{abs}(r - n)$.

Input

The input consists of:

- one line with the positive integer n ($1 \leq n \leq 10^{18}$).

Output

Output the closest number to n with exactly one significant (non-zero) figure. If there are two equally-close answers, print the larger one.

Sample Input 1	Sample Output 1
`150`	`200`

Sample Input 2	Sample Output 2
`4`	`4`

Sample Input 3	Sample Output 3
`33471234512345`	`30000000000000`

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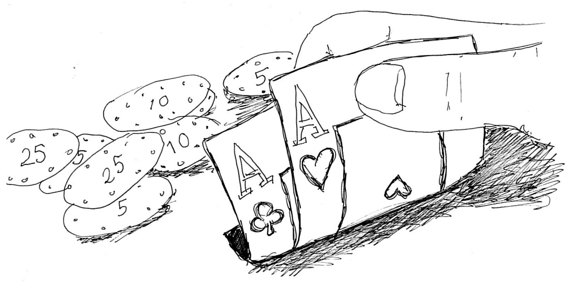
Problem C
Crooked Dealing



This week you started a flashy new job in Leeds as a poker dealer. Your task will be to hand out the cards for games. The base pay is not particularly good, but you found out that you can earn a lot from tips if you deal the cards well.

The most generous customers prefer that their opponents at the table don't receive any pairs of cards with the same number; so to keep them happy you will make sure this never happens.

You already know the numbers on every card in the pile , and the number of cards any player must have in their hand. Find how many hands you can make at once without introducing a pair.

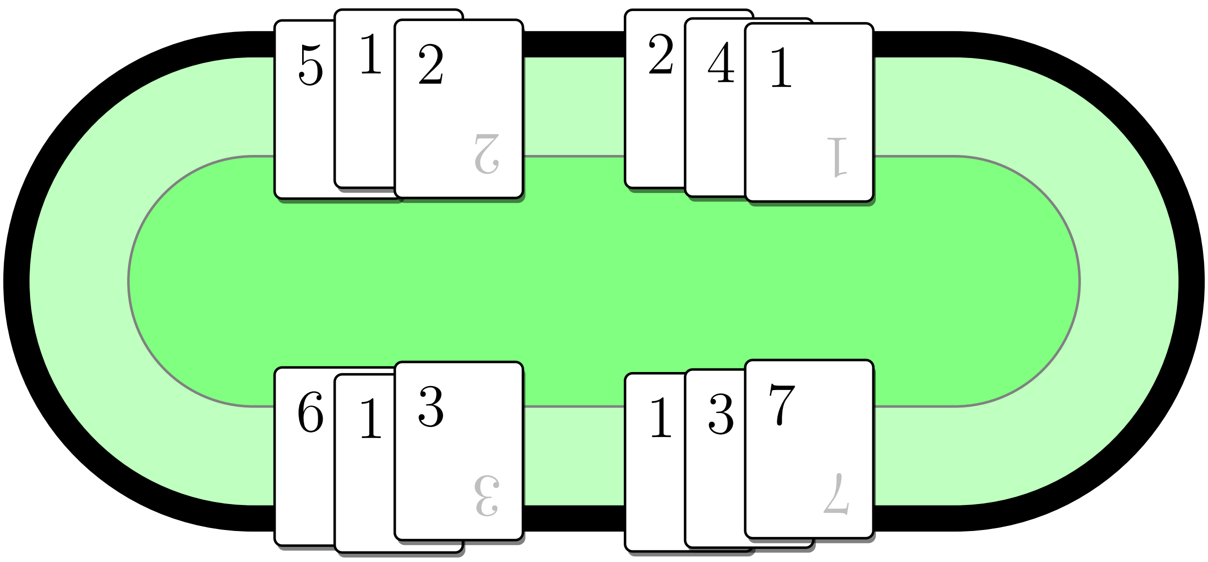


Figure C.1: Illustration of a solution to Sample Input 2.

Input

The input consists of:

- A line with two integers n and h ($1 \leq h \leq n \leq 10^6$), the number of cards in the pile, and the number of cards in a hand.
- A line with n integers $v_1, \ldots, v_n$ ($1 \leq v_i \leq 10^6$ for all i), the numbers on the cards in no particular order.

Output

If it is not possible to make any hands at all without introducing a pair, output `impossible`.

Otherwise, output k lines (where k is the maximum possible number of players) each containing h numbers from the input. You may not use any number any more times than it appears in v.

If there are multiple answers with a maximum value of k, you may output any one of them.

Sample Input 1	Sample Output 1
6 3 1 2 1 2 3 4	1 2 4 1 2 3

Sample Input 2	Sample Output 2
14 3 3 4 1 1 1 2 3 1 2 1 1 5 6 7	6 1 3 2 4 1 5 1 2 1 3 7

Sample Input 3	Sample Output 3
8 5 1 1 2 2 3 3 4 4	impossible

Problem D
Dome Construction

The world's largest indoor water park is built inside a hemispherical dome that was once used as an aircraft hangar. The park attracts more than $10\,000$ visitors per day and is big enough that it even has its own tropical micro-climate with clouds forming inside.

Management would like to expand business operations by opening another branch in the dome of your local cathedral. The micro-climate is a key selling point, so to really capitalise on the cathedral they asked you to expand the dome's radius so that it contains at least a given number of clouds. A cloud is contained if its centre is on or inside the boundary of the dome.

You are a cloud engineer by trade, and hence a competent meteorologist. You already identified several potential clouds close by and plotted them in three dimensions relative to the centre of the current structure. In order to capture enough of them, how large do you need to make the radius of the dome?

Input

- The first line contains the number of clouds you found, n, and the number that must be contained, k, respectively ($1 \le k \le n \le 10^5$).
- The next n lines each contain three real numbers x_i, y_i, z_i, the coordinates of the ith cloud relative to the centre of the dome ($0 \le |x_i|, |y_i|, |z_i| \le 10^6$). Every cloud has a non-negative y-coordinate.

Output

Output the minimum radius of the dome required to enclose at least k points. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1	Sample Output 1
5 3 -4 2 1 2.1 3 5 1.2 1 -1 -2.2 3 2 1 0 2.1	4.22374242

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Problem E
Estate Agent

Rupert makes a living as the only real estate agent in a small town in England. He asks for 5% commission for every house that he sells.

Rupert organises one big auction per year. Every family (numbered from 1 to n) must participate in this action, although making or an accepting an offer is optional. Everyone puts in bids for the houses they would like to move to, provided they can sell their current house at the same time.

This is a very transparent process–Rupert can see exactly how much commission he will make if he accepts the right buyers' offers on behalf of the sellers. He may discard some offers from buyers in order to drive up the overall commission. In fact, he might even decide to discard all of the offers from one family and let them stay in their current home, if it makes more money for him.

Find the maximum commission Rupert can make if he discards offers optimally.

Input

The input consists of:

- one line containing two integers n and m ($1 \le n \le 150$, $0 \le m \le n \times (n-1)$), the number of families on the market and the number of offers made.
- m lines, describing the offers.

  The ith such line contains three integers f_i, h_i and a_i ($1 \le f_i, h_i \le n$, $f_i \ne h_i$, $0 \le a_i \le 10^6$), the family making the offer, the family that owns the house that the offer is for and the amount offered.

  No family makes more than one offer to the same house.

Output

Output how much Rupert will earn via commissions if he discards offers optimally. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1	Sample Output 1
4 5 1 2 3 2 3 9 3 1 5 3 2 11 4 1 6	1.0

Sample Input 2	Sample Output 2
4 5 1 2 15 2 3 9 3 1 5 3 2 11 4 1 6	1.45

Problem F
Feeding Seals

You are in charge of feeding the seals in the Welsh Mountain Zoo. This involves purchasing buckets of fish and allocating them to volunteers to trek into the enclosure and distribute fairly to the blubbery residents.

The buckets of fish are already set out. Each volunteer can be assigned to carry either one or two of these buckets, as long as the combined weight of the buckets is small enough.

How many volunteers will you need to distribute all of the fish in one trip?

Input

- The first line contains the number of buckets to be delivered, n ($1 \leq n \leq 10^5$), and the integer carrying capacity of a volunteer, c ($1 \leq c \leq 10^9$).
- The second line contains the integer weights of each of the n buckets, $w_1 \ldots w_n$ ($1 \leq w \leq c$).

Output

Output the minimum number of volunteers required to deliver all of the buckets of fish.

Sample Input 1	Sample Output 1
4 100 44 35 66 67	3

Sample Input 2	Sample Output 2
1 10 7	1

Sample Input 3	Sample Output 3
3 12 10 5 6	2

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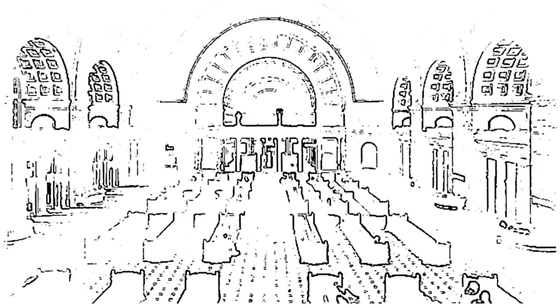
Problem G
Grand Central Station



The city you live in just finished construction of its new transport network, PlusRail. There are n stations and exactly one way to get between any given pair of them; this is because there are only $n-1$ direct station:station connections. In other words, the network forms a tree.

You have been hired to put together the signage for each of the stations which shows where on the network a passenger is with a big arrow pointing to the bright red station in the centre.

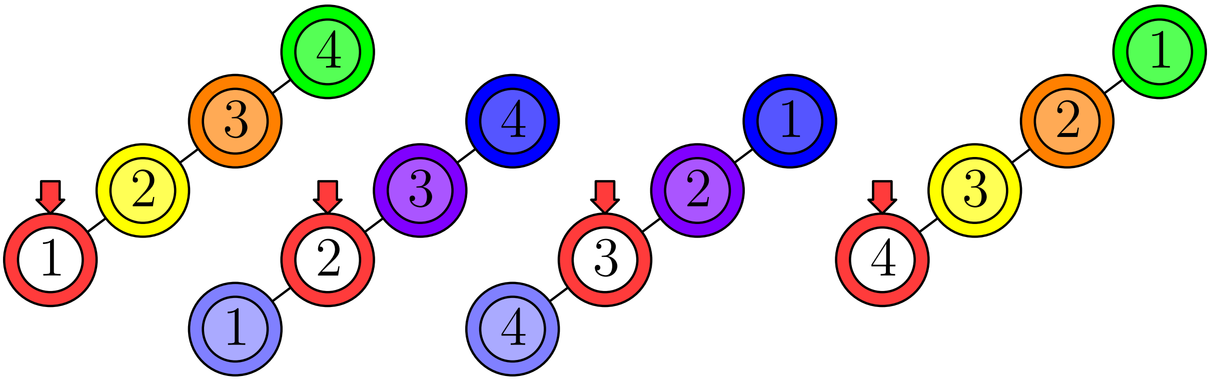


Figure G.1: Illustration of Sample Input 1 and how two designs are reused four times, with the labels painted at different places.

Because the drawings of the network are fairly crude, it is actually possible that you could use the same sign in more than one station, and just write a different permutation of labels for the station names.

If you want to make signage for the whole network, what is the minimum number of unique designs you will need?

Input

- The first line of input contains the number of stations, n ($1 \le n \le 3 \times 10^5$).
- The following $n-1$ lines each contain two distinct vertex indices a and b ($1 \le a, b \le n$) indicating that there is a direct route between these stations.

Output

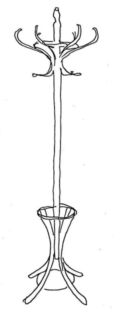
Output the minimum number of map designs that can be made, such that for any station at least one of these map designs can be re-labelled such that this station is in the centre.

Sample Input 1	Sample Output 1
4 1 2 2 3 3 4	2

Sample Input 2	Sample Output 2
11 1 2 2 3 3 4 4 5 4 6 4 7 5 10 10 9 10 8 7 11	10

Sample Input 3	Sample Output 3
7 7 1 7 2 3 2 7 4 5 4 6 5	7

Problem H
Hat Stand



You own several hats, some of which you wear more than others. Because wearing all n at once would be impractical, you store the spares on a rack with $n - 1$ hooks. The first hook is a half metre from the door, the second one metre, the third a metre-and-a-half, and so on. This means that walking from and to the door to the ith hook involves i metres of walking.

Tonight you will need to make a series of public appearances, each time wearing headgear appropriate to the role. When you come back from one engagement, you will take the hat you need from its hook and exchange it with the one you had been wearing before. This means that hats can move around as the night goes on.

Given the plan for tonight, and assuming you are already wearing the first one, what is the best way to arrange the hats on the rack before setting off so as to minimise the number of metres walked?

Input

- The first line of input contains two integers: c and n ($1 \leq c, n \leq 10^5$), the number of hats in total and the length of the itinerary respectively.
- The second line of input contains n integers: $v_1 \ldots v_n$ ($1 \leq v \leq c; v[i] \neq v[i-1]$), the order of hats you need to put on. You are already wearing the first one, and you never need to wear the same hat twice consecutively.

Output

Output the minimum number of metres you need to walk, once you have optimised your hat rack.

Next, output $c - 1$ integers: an initial ordering of the hat rack that gives minimum walking distance, starting from place 1. This should include exactly once every hat except the first.

If there are multiple correct answers, you may output any one of them.

Sample Input 1	Sample Output 1
3 5 1 3 2 3 2	5 2 3

Sample Input 2	Sample Output 2
5 9 1 4 2 3 5 2 5 2 5	14 3 5 4 2

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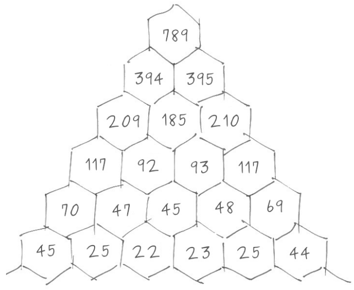
Problem I
Integral Pyramid



Pascal's triangle is a marvel of the combinatorical world, and what's more you can easily build one for yourself at home.

The lowest row has n numbers. The next row is staggered and has $n - 1$ numbers, where the ith is the sum of the ith and the $i + 1$th on the previous row.

You can choose any positive integers for the lowest row, but the single cell on the top row needs to be equal to a given x. Is this possible?

Input

- The only line contains the number of rows, n ($1 \leq n \leq 20$), and the value needed at the top, x ($1 \leq x \leq 10^9$).

Output

If a pyramid can be constructed, output all of the numbers on each row, starting from the top. Every number must be greater than or equal to 1.

Otherwise, output `impossible`.

Sample Input 1
```
3 15
```

Sample Output 1
```
15
8 7
3 5 2
```

Sample Input 2
```
6 789
```

Sample Output 2
```
789
394 395
209 185 210
117 92 93 117
70 47 45 48 69
45 25 22 23 25 44
```

Sample Input 3
```
20 1
```

Sample Output 3
```
impossible
```

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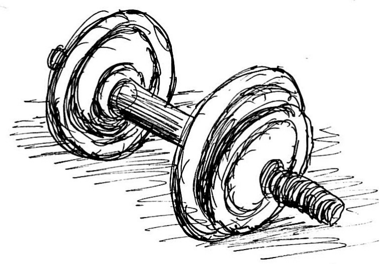
Problem J
Jammed Gym



You are at the fitness centre to run through your exercise programme. You must use the kinds of exercise machine in an order precisely dictated by the programme, although there may be more than one instance of a machine.

You start at the centre of a unit circle around which the exercise stations are arranged. You can walk directly between any two points in the circle, and you may also visit the same point multiple times. See Figure J.1 below for an example.

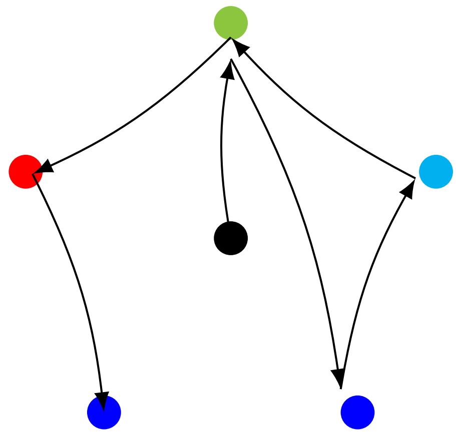


Figure J.1: Illustration of Sample Input 1. Types of machine: $[1, 2, 4, 1, 3, 2]$

Exercise is an important and noble endeavour, but in today's busy world we must strive for efficiency in everything we do. Find the most efficient way of visiting exercise stations that matches the order given.

Input

- The first line of input contains the number of exercises in the programme, n ($1 \leq n \leq 100$).
- The second line of input contains n space-separated integers each denoting the type of an item on the programme t ($1 \leq t_i \leq 100$). There will always be at least one station for each programme in this list.
- The third line of input contains the number of stations, m ($1 \leq m \leq 100$).
- The fourth line of input contains m space-separated integers each denoting the type of a station q ($1 \leq q_i \leq 100$).

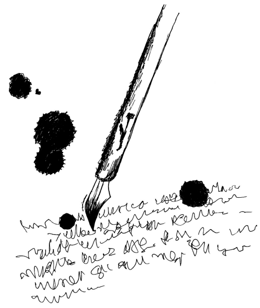
Output

Output the minimum distance you will need to walk. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1	Sample Output 1
6 1 2 4 1 3 2 5 1 4 2 2 3	7.604395

Sample Input 2	Sample Output 2
5 4 2 1 3 1 6 1 2 1 3 1 4	5.732051

Problem K
Knocked Ink



You knocked over the inkwell for the team fountain pen... Now spots of ink are beginning to form on the page and spread out. This is really going to hamper your speed at writing up programming contest solutions.

The ink spreads out by forming infinitesimally-small blots on the page. A blot that appears at time t seconds after the incident grows in radius smoothly, at a rate of 1cm per second, and may eventually overlap with other blots on the page.

At first the page is still usable, but when the combined size of the ink blots grows large enough, you will have to abandon your work and find another piece of paper upon which to type up solutions.

How long will it take for this to happen?

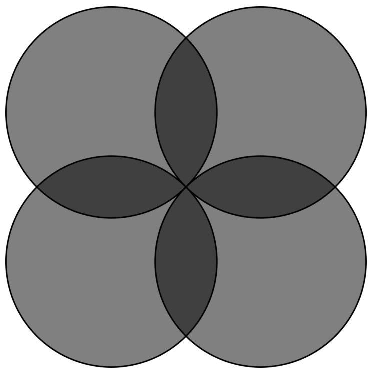


Figure K.1: Illustration of Sample Input 1.

Input

The input consists of:

- one line with the number of inkblots, n ($1 \leq n \leq 100$), and the real-valued total area of ink in square centimetres at which the paper must be abandoned, a ($1 \leq a \leq 10^9$).
- n further lines, each with the x and y coordinates in centimetres of an ink blot ($-10^6 \leq x, y \leq 10^6$) and the time in seconds at which the blot first appears, t ($0 \leq t \leq 10^6$).

Output

Output the time in seconds at which the ink blots cover exactly a square centimetres of the infinitely-large page. Your answer must be accurate to an absolute or relative error of 10^{-6}.

Sample Input 1

```
4 20.566371
0.0 0.0 0.0
0.0 2.0 0.0
2.0 0.0 0.0
2.0 2.0 0.0
```

Sample Output 1

```
1.4142135624
```

Sample Input 2

```
2 785.398163397
-50 0 20
50 0 30
```

Sample Output 2

```
35
```

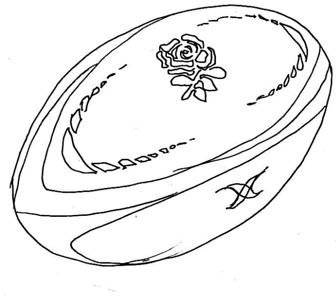
Sample Input 3

```
5 10000
0 0 0
0 0 1
0 0 2
10 0 1
0 -5 2
```

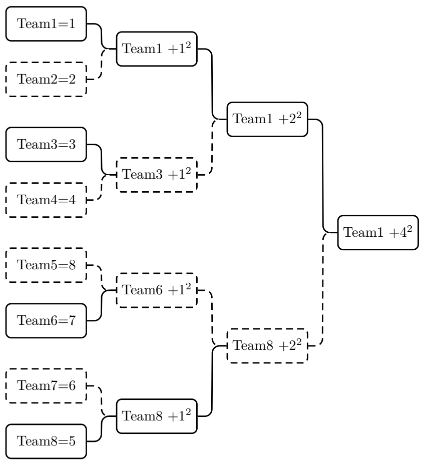
Sample Output 3

```
53.3322048
```

Problem L
Low Effort League



The teams in your local rugby league aren't particularly good, but they make up for it in enthusiasm. We are going to organise a single-elimination knockout tournament between them, where the 2^n teams play n rounds. In each round, the $2i + 1$th remaining team pairs up with the $2i + 2$th team and one or the other team is eliminated.



Each team has a scalar skill level. In the normal course of things, a team with higher skill level will always beat a team with lower skill level. However, training plays a part too: if one team studies another, learns its techniques, and trains against them, it can win.

The number of hours a team with skill a must train to beat a team with skill b (where $a \leq b$) is $|b - a|^2$. This training only affects that one game (it does not transfer to other teams).

You would quite like your favourite team to win the tournament. If you take complete control over how every team trains, you can always make this happen. What is the minimum number of hours needed, in total across all teams, in order for your team (team 1) to win?

Input

The input consists of:

- one line containing the integer r ($1 \leq r \leq 14$), the number of rounds in the tournament.
- one line with 2^r integers $s_1 \ldots s_{2^s}$ ($0 \leq s_i \leq 10^6$ for each i), where s_i is the skill level of the ith team.

Output

Output the smallest number of hours needed for team 1 to win the tournament.

Sample Input 1

```
1
50 40
```

Sample Output 1

```
0
```

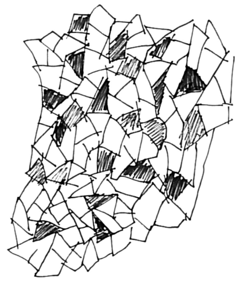
Sample Input 2

```
3
1 2 3 4 8 7 6 5
```

Sample Output 2

```
28
```

Problem M
Mosaic Mansion



A mosaic is a picture made from square tiles arranged in a grid, at least for today's purposes.

We would like to make a mosaic with exactly the same number of tiles of each colour. We will do this by taking an existing design and removing some of the rows from it.

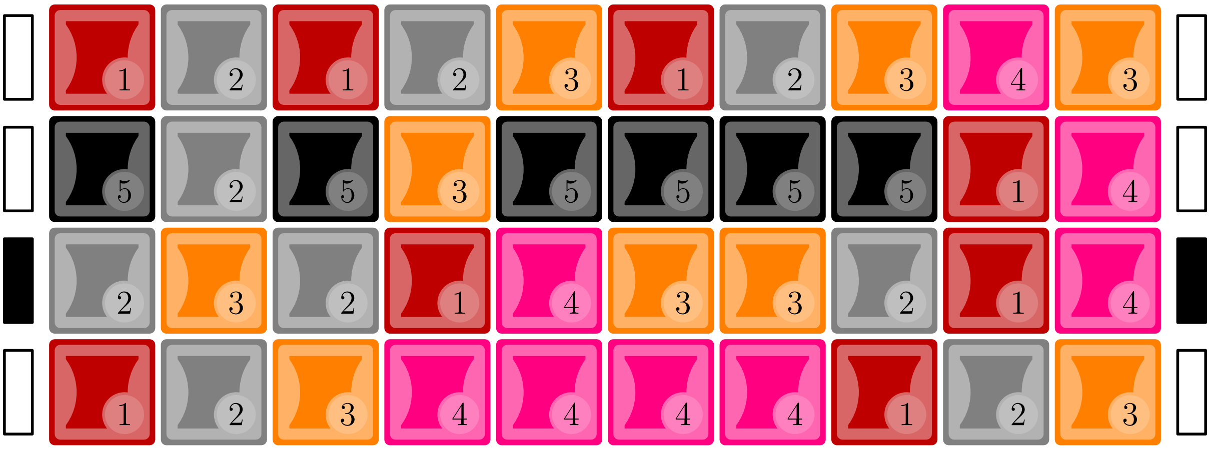


Figure M.1: Illustration of a solution to Sample Input 1. The three rows annotated with white can be kept, giving 6 of each colour of tile.

What is the greatest number of rows we can keep?

Input

- The first line of input contains the number of rows, n ($1 \leq n \leq 40$), the number of columns, m ($1 \leq m \leq 10^5$), and the number of colours, c ($1 \leq c \leq 10^5$) in the mosaic respectively.
- Each of the next n lines contains m colours of cells $p_1 \ldots p_m$ ($1 \leq p \leq c$).

Output

Output the greatest number of rows that can be kept while keeping equal representation for each colour in the input, or 0 if no rows can be kept.

Sample Input 1	Sample Output 1
4 10 5 1 2 1 2 3 1 2 3 4 3 5 2 5 3 5 5 5 5 1 4 2 3 2 1 4 3 3 2 1 4 1 2 3 4 4 4 4 1 2 3	3

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